“Timing Vertical Relationships”

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Research questions

In a *growing market with uncertain demand*, where one or several firms need investing in a *key input* to start operations, we adapt the standard analysis of *vertical* relationships (“separation”) to investment decisions.

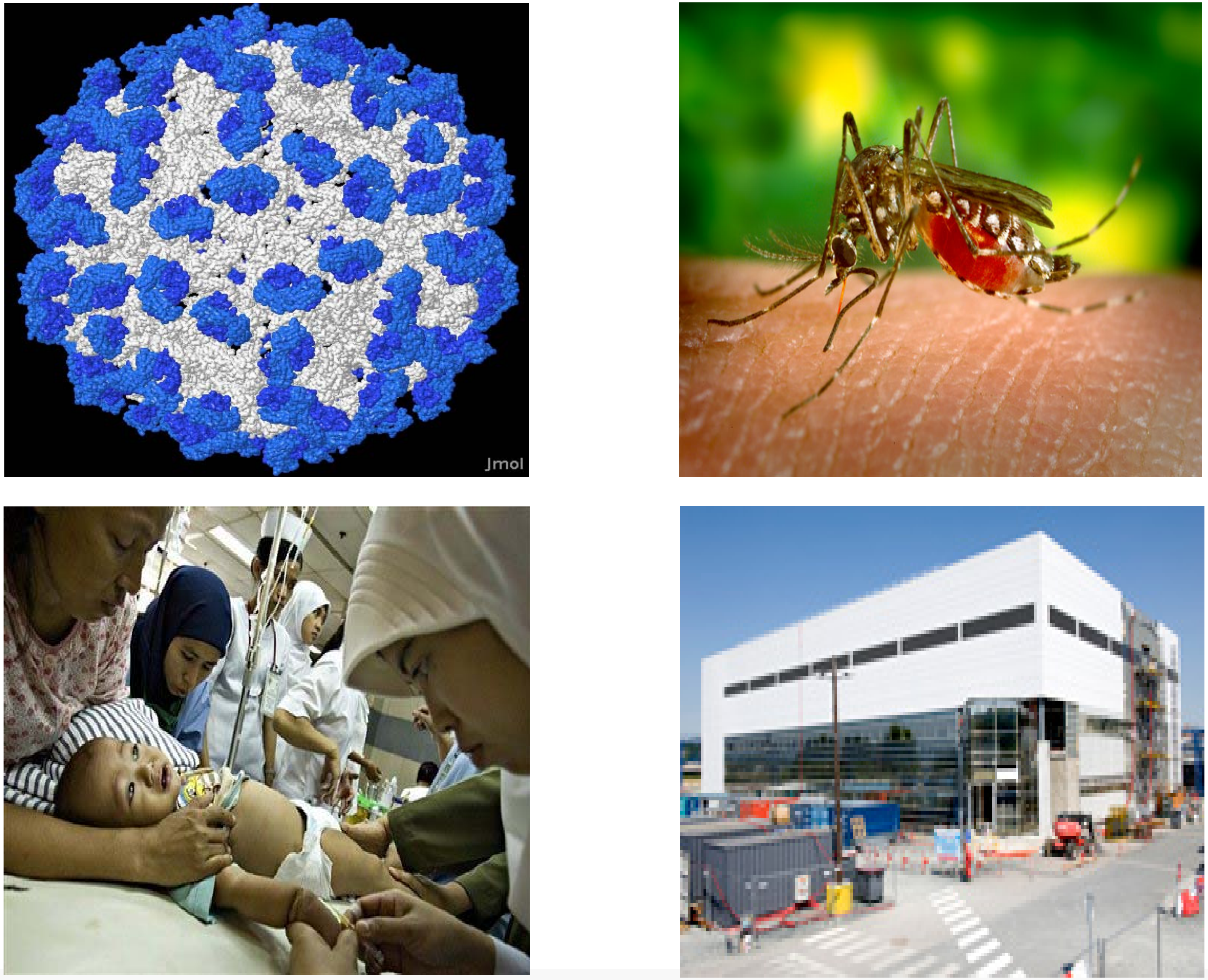
What if the intermediate market is *not* competitive ?

What is the impact of separation on the *cost* and *timing* of investments ?

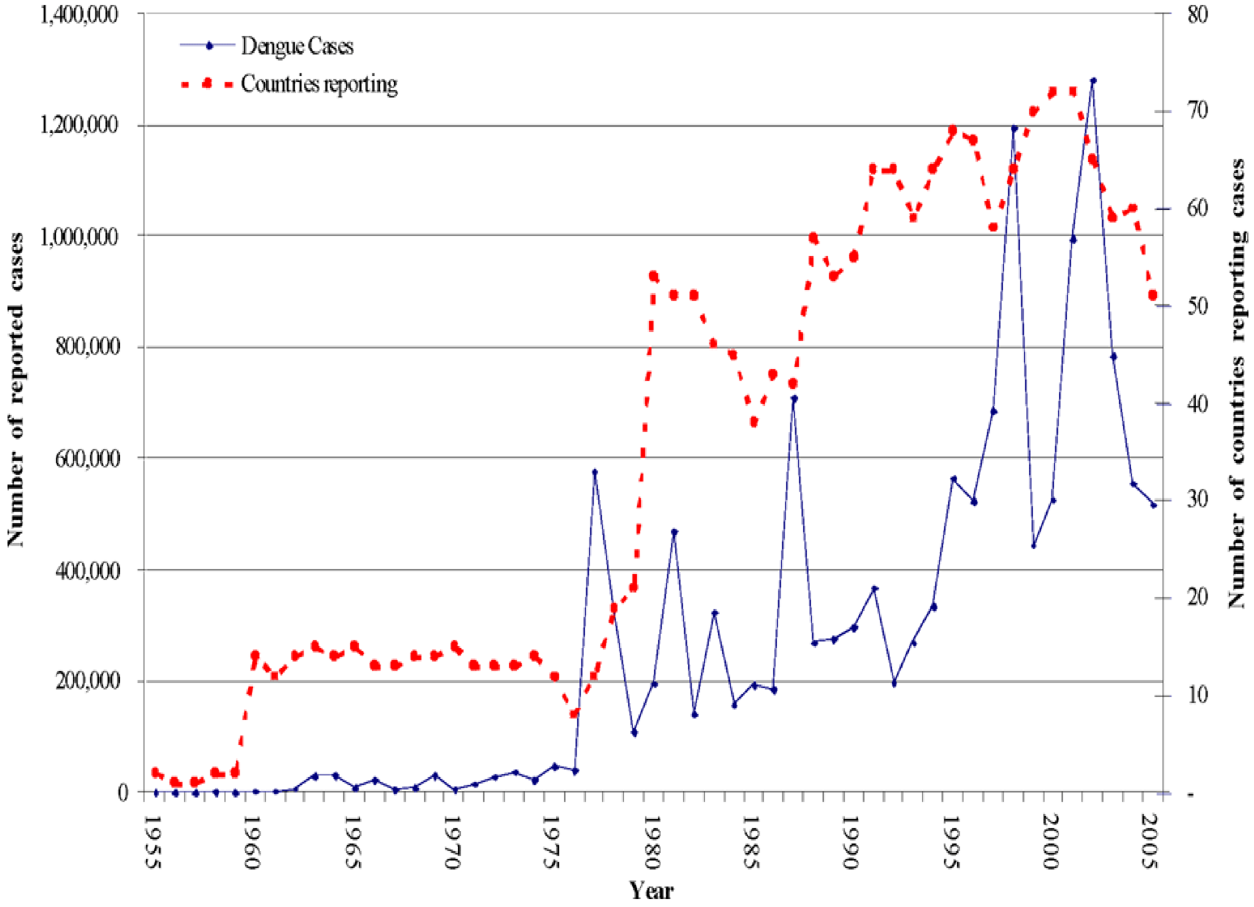
On firm *values*? With *one* or *several* downstream firms ?

Sensitivity to demand parameters (growth, volatility) ?

▶ 1. Motivation
2. The Model
3. Integration/Separation
4. Vertical Restraints
5. Two Buyers
6. Final Remarks



sources: http://www.sanofi-aventis.com, http://www.rcsb.org



source: www.tropika.net/svc/review/061001-Dengue_Burden_of_disease

lyophilizator
supplier

↓ ↘

sanofi pasteur
La division vaccins du Groupe sanofi-aventis.

gsk GlaxoSmithKline

Approach

We formalize investment decisions in a key technology which is needed to serve a continuously changing and uncertain final demand.

We compare "integration" (1 firm) with "separation" (1 technology supplier + 1 downstream firm) and "preemption" (1 technology supplier + 2 downstream firms).

Main Results

(i) Separation introduces distortions in the cost of technology, hence in the investment timing.

(ii) Vertical restraints, as adapted from the static framework, may eliminate the distortions.

(iii) Preemption and price discrimination restore private efficiency.

Specifications (1/3)

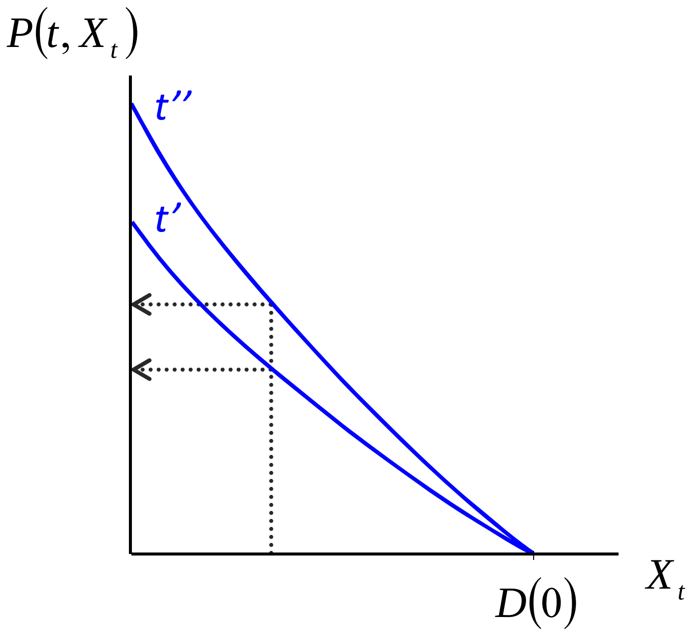
Model specifications similar to models of real option games (Smit and Trigeorgis, PUP 2004; Mason and Weeds, IJIO 2009; Boyer, Lasserre and Moreaux, CIRANO WP 2010), *plus* an upstream input supplier which prices with market power:

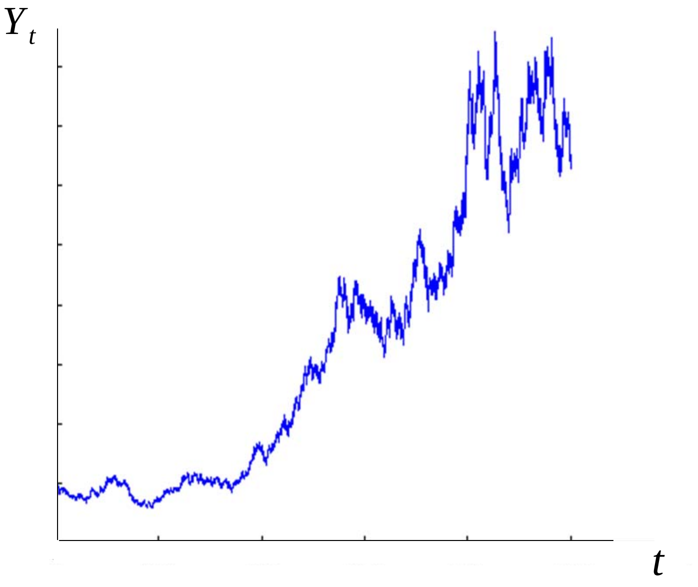
Specifications (1/3)

Model specifications similar to models of real option games (Smit and Trigeorgis, PUP 2004; Mason and Weeds, IJIO 2009; Boyer, Lasserre and Moreaux, CIRANO WP 2010), *plus* an upstream input supplier which prices with market power:

- continuous time
- a key input is needed (no resale value)
- changing demand with deterministic positive drift + stochastic term
- integration (benchmark): the firm produces the input
- separation: one upstream firm, one (or two) downstream firm(s)
- two-level strategic interaction:
  - ◦ in investment timing (max. of net expected value) = “long run”
  - ◦ in price or quantity (max. of gross instant. profits) = “short run”

Specifications (2/3)





$$P(t, X_t) = Y_t D^{-1}(X_t)$$

random shock

output

Wiener process

$$dY_t = \alpha Y_t dt + \sigma Y_t dZ_t$$

growth rate

volatility

Specifications (3/3)

$\boxed{y = Y_t}$: current “market size”

$Y_t \pi_M$: instantaneous monopoly profits

$Y_t \pi_D$: instantaneous duopoly profits

$\boxed{y_i}$: investment trigger

$i =$ D for “downstream”
P for “preemption”
F for “follower”
L for “leader”

I : “true” cost of investment

p_U : price charged upstream to single buyer

p_{U_F} : price charged upstream to “follower”

p_{U_L} : price charged upstream to “leader”

Integration (benchmark)

A single firm produces the discrete input and decides at what threshold y_i to enter the market whose current size is $y \leq y_i$.

Its value is function is

$$\boxed{V\left(y, y_i, I\right) = \left(\frac{y}{y_i}\right)^{\beta} \left(\frac{\pi_M}{r-\alpha} y_i - I\right)}$$

all $y \leq y_i$, with $\beta = \frac{1}{2} - \frac{\alpha}{\sigma^2} + \sqrt{\left(\frac{\alpha}{\sigma^2} - \frac{1}{2}\right)^2 + \frac{2r}{\sigma^2}} > 1$. The value-maximizing investment trigger is

$$\boxed{y^* = \frac{\beta}{\beta - 1} \frac{r-\alpha}{\pi_M} I}$$

The value of the firm that invests at y^* is

$$\boxed{V\left(y, y^*, I\right) = \frac{I}{\beta - 1} \left(\frac{y}{y^*}\right)^{\beta}.}$$

Separation

An upstream firm U sells the input at price $p_U \geq I$. Given p_U, a downstream firm D observes the final market shock and decides when to enter, at y_i.

The downstream value function is

$$\boxed{V\left(y, y_i, p_U\right) = \left(\frac{y}{y_i}\right)^{\beta}\left(\frac{\pi_M}{r-\alpha} y_i - p_U\right)}$$

all $y \leq y_i$. The downstream value-maximizing investment trigger is

$$\boxed{y_D\left(p_U\right) = \frac{\beta}{\beta-1}\frac{r-\alpha}{\pi_M} p_U.}$$

The upstream firm's value function is

$$\boxed{W\left(y, p_U\right) = \left(\frac{y}{y_D(p_U)}\right)^{\beta}\left(p_U - I\right)}$$

all $y \leq y_D$. Given $y_D\left(p_U\right)$, the upstream firm maximizes $W\left(y, p_U\right)$ by charging

$$\boxed{p_U^* = \frac{\beta}{\beta-1} I.}$$

Proposition 1 *In the separated case, there is a unique equilibrium characterized by*

$$y_D^* = \left(\frac{\beta}{\beta - 1}\right)^2 \frac{r - \alpha}{\pi_M} I \quad > \quad y^* = \frac{\beta}{\beta - 1} \frac{r - \alpha}{\pi_M} I$$

Proposition 1 *In the separated case, there is a unique equilibrium characterized by*

$$y_D^* = \left(\frac{\beta}{\beta-1}\right)^2 \frac{r-\alpha}{\pi_M} I \text{ and } p_U^* = \frac{\beta}{\beta-1} I > I$$

Proposition 1 *In the separated case, there is a unique equilibrium characterized by*

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The firm values in the separated equilibrium case are

$$V\left(y, y_D^*, p_U^*\right) = \frac{\beta}{(\beta-1)^2}\left(\frac{y}{y_D^*}\right)^\beta I \text{ and } W\left(y, p_U^*\right) = \frac{1}{\beta-1}\left(\frac{y}{y_D^*}\right)^\beta I.$$

Relative value upstream/downstream:

$$\frac{W\left(y, p_U^*\right)}{V\left(y, y_D^*, p_U^*\right)} = \frac{\beta - 1}{\beta} < 1, \text{ increasing in } \beta.$$

Relative joint value under separation and integration:

$$\frac{V\left(y, y_D^*, p_U^*\right) + W\left(y, p_U^*\right)}{V\left(y, y^*, I\right)} = \left(1 + \frac{\beta}{\beta - 1}\right)\left(\frac{\beta}{\beta - 1}\right)^{-\beta} < 1, \text{ decreasing in } \beta.$$

Upstream distortion measure:

$$L_U \equiv \frac{p_U^* - I}{p_U^*} = \frac{1}{\beta}.$$

Donwstream distortion measure:

$$L_D \equiv \frac{y_D^* - y^*}{y_D^*} = \frac{1}{\beta}.$$

Impact of a change in α (growth) and/or σ (volatility)?

$$\frac{\mathrm{d}\beta}{\mathrm{d}\alpha} = \frac{-\beta}{\left(\beta - \left(\frac{1}{2} - \frac{\alpha}{\sigma^2}\right)\right)\sigma^2} < 0$$

$$\frac{\mathrm{d}\beta}{\mathrm{d}\sigma} = \frac{-2\left(r - \alpha\beta\right)}{\left(\beta - \left(\frac{1}{2} - \frac{\alpha}{\sigma^2}\right)\right)\sigma^3} < 0$$

$(\lim_{\alpha\to r}\beta = \lim_{\sigma\to\infty}\beta = 1)$

Proposition 2 *The industry value is lower under separation than under integration. The distortion in firm decisions, as measured by* L_U *and* L_D*, is increasing in market growth rate and volatility*.

Comparative statics: growth

Let $V^* \equiv V\left(y, y_D^*, p_U^*\right)$, all $y \leq y_D^*$.

By the envelope theorem, $\frac{\partial V}{\partial y_D^*} = 0$, so that

$$\frac{\mathrm{d}V^*}{\mathrm{d}\alpha} = \underbrace{\frac{\partial V^*}{\partial \alpha}}_{>0} + \left(\underbrace{\frac{\partial V^*}{\partial \beta}}_{<0} + \underbrace{\overbrace{\frac{\partial V}{\partial p_U^*}}^{<0} \overbrace{\frac{\partial p_U^*}{\partial \beta}}^{<0}}_{>0} \right) \underbrace{\frac{\partial \beta}{\partial \alpha}}_{<0} = V^* \left(\frac{\beta}{r-\alpha} + \left(\ln \frac{y}{y_D^*} + \frac{1}{\beta} \right) \frac{\partial \beta}{\partial \alpha} \right) > 0.$$

real option
indirect effect

price
indirect effect

Comparative statics: volatility

Let $V^* \equiv V\left(y, y_D^*, p_U^*\right)$, all $y \leq y_D^*$.

By the envelope theorem, $\frac{\partial V}{\partial y_D^*} = 0$, so that

$$\frac{\mathrm{d}V^*}{\mathrm{d}\sigma} = \left(\underbrace{\frac{\partial V^*}{\partial \beta}}_{<0} + \underbrace{\overbrace{\frac{\partial V}{\partial p_U^*}}^{<0} \overbrace{\frac{\partial p_U^*}{\partial \beta}}^{<0}}_{>0} \right) \underbrace{\frac{\partial \beta}{\partial \sigma}}_{\leq 0} = V^* \left(\ln \frac{y}{y_D^*} + \frac{1}{\beta} \right) \frac{\partial \beta}{\partial \sigma}.$$

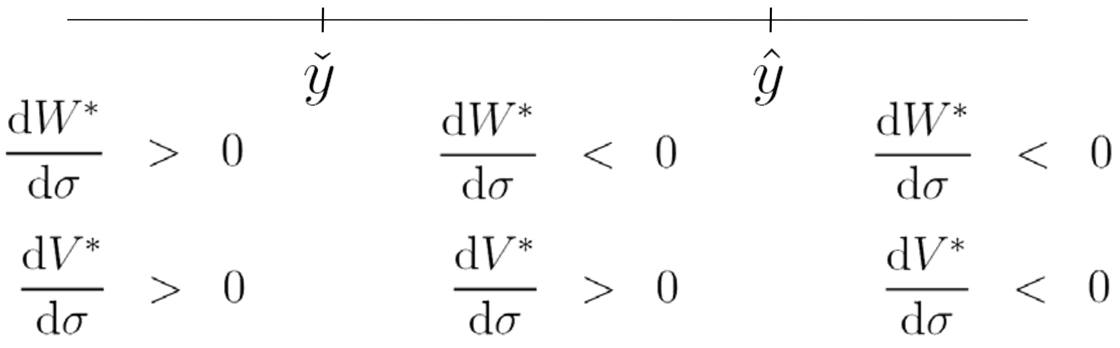
real option
indirect effect

price
indirect effect

The crossover occurs at $y_D^* \exp\left(-\frac{1}{\beta}\right) \equiv \hat{y} < y_D^*$.

Proposition 3 *In the separated case, a higher growth rate or more volatility increase the upstream price and the downstream trigger. A higher growth rate increases upstream and downstream values, with* $0 < \varepsilon_{W^*/\alpha} < \varepsilon_{V^*/\alpha}$. *The effect of higher volatility on firm values depends on the market size:*

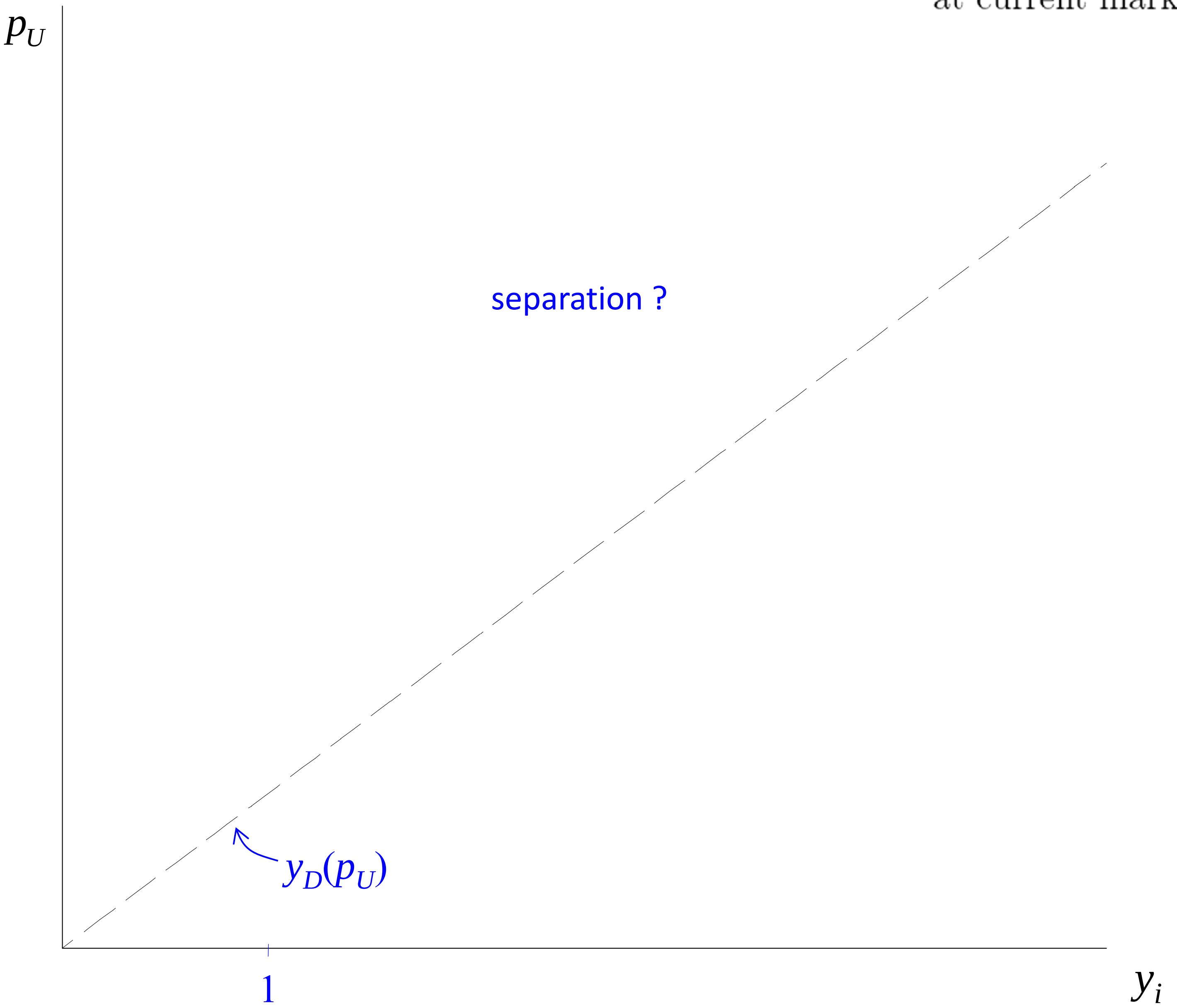
$$\begin{cases} 0 < \varepsilon_{W^*/\sigma} < \varepsilon_{V^*/\sigma} & \text{if } y \leq \check{y}; \\ \varepsilon_{W^*/\sigma} < 0 < \varepsilon_{V^*/\sigma} & \text{if } \check{y} < y < \hat{y}; \\ \varepsilon_{W^*/\sigma} < \varepsilon_{V^*/\sigma} < 0 & \text{if } \check{y} \leq y. \end{cases}$$

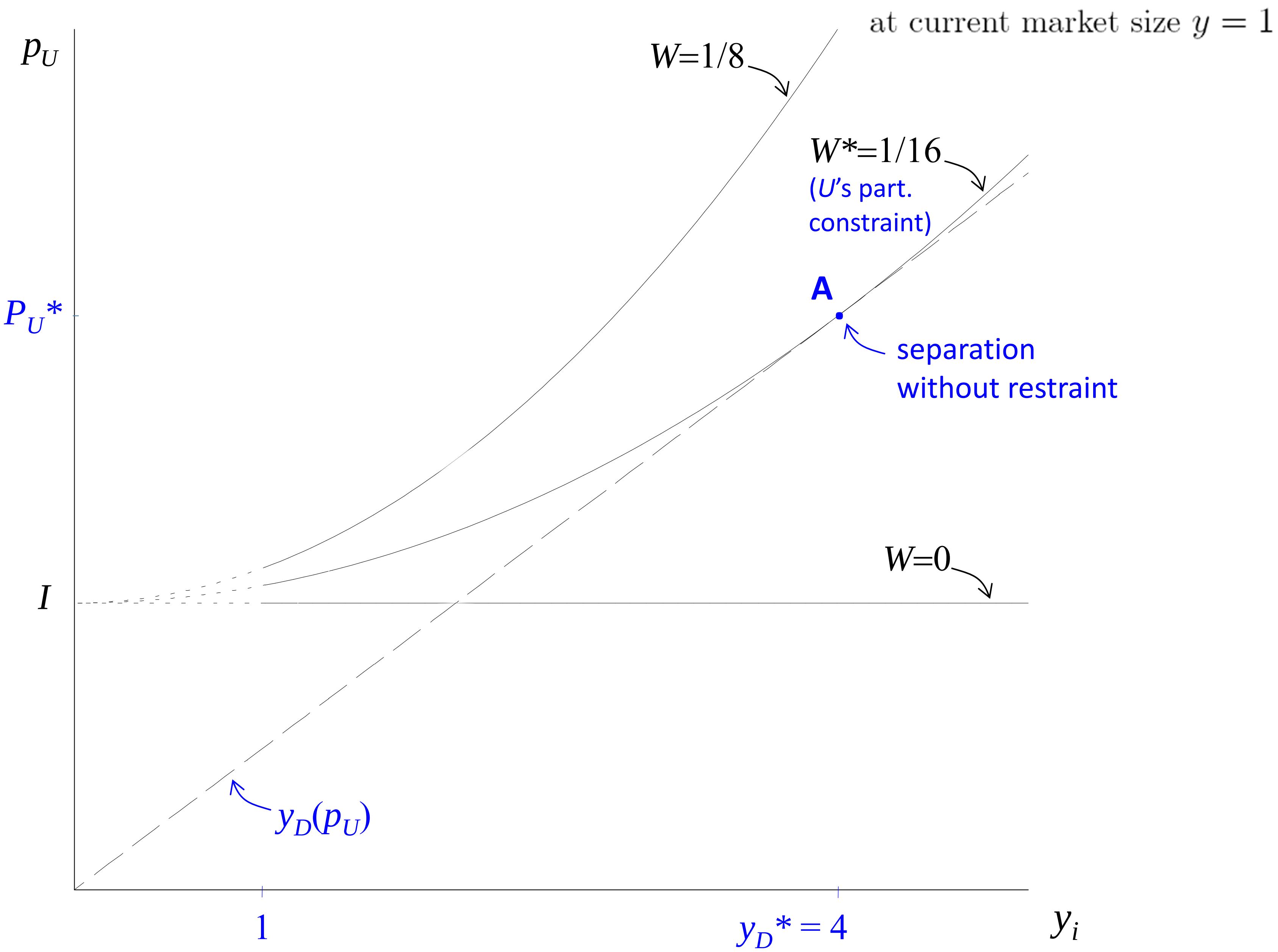


$$\frac{\mathrm{d}W^*}{\mathrm{d}\sigma} > 0 \qquad \frac{\mathrm{d}W^*}{\mathrm{d}\sigma} < 0 \qquad \frac{\mathrm{d}W^*}{\mathrm{d}\sigma} < 0$$

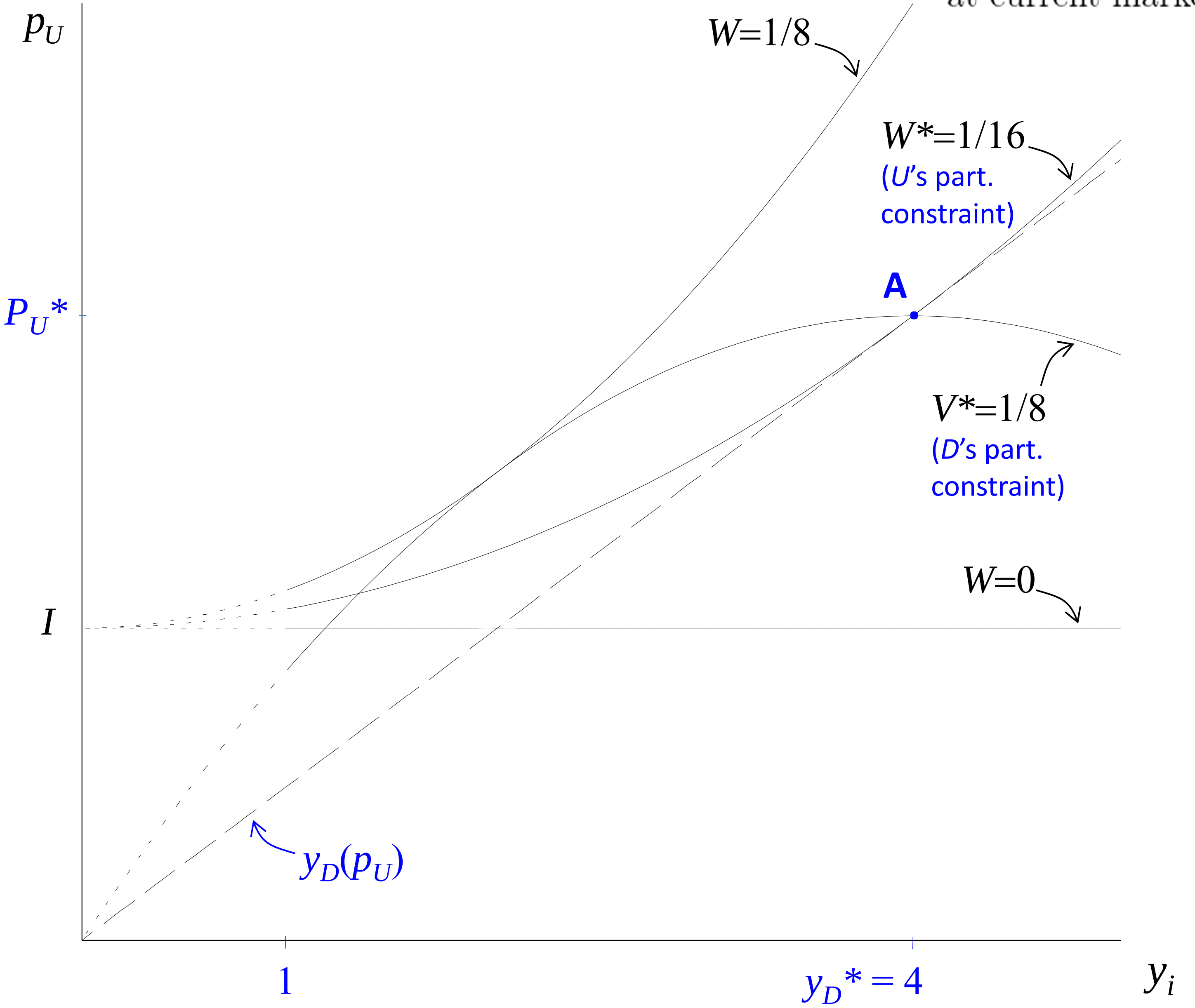
$$\frac{\mathrm{d}V^*}{\mathrm{d}\sigma} > 0 \qquad \frac{\mathrm{d}V^*}{\mathrm{d}\sigma} > 0 \qquad \frac{\mathrm{d}V^*}{\mathrm{d}\sigma} < 0$$

at current market size $y = 1$




at current market size $y = 1$
p_U
W=1/8
W^*=1/16
(U's part. constraint)
A
$P_U{}^*$
separation without restraint
W=0
I
$y_D(p_U)$
1
$y_D{}^* = 4$
y_i

at current market size $y = 1$



Analogous to RPM upstream problem:

$$\max_{y_i, p_U} W\left(y, y_i, p_U\right)$$
$$\text{s.t.} \qquad p_U \leq \overline{p}_U\left(y_i\right),$$
$$p_U \geq \underline{p}_U\left(y_i\right),$$

all $y \leq y^*$, where $\underline{p}_U\left(y\right)$ is defined by $W\left(y, y_D^*, p_U^*\right) = W\left(y, y_i, \underline{p}_U\left(y\right)\right)$, and $\overline{p}_U\left(y\right)$ by $V\left(y, y_D^*, p_U^*\right) = V\left(y, y_i, \overline{p}_U\left(y\right)\right)$.

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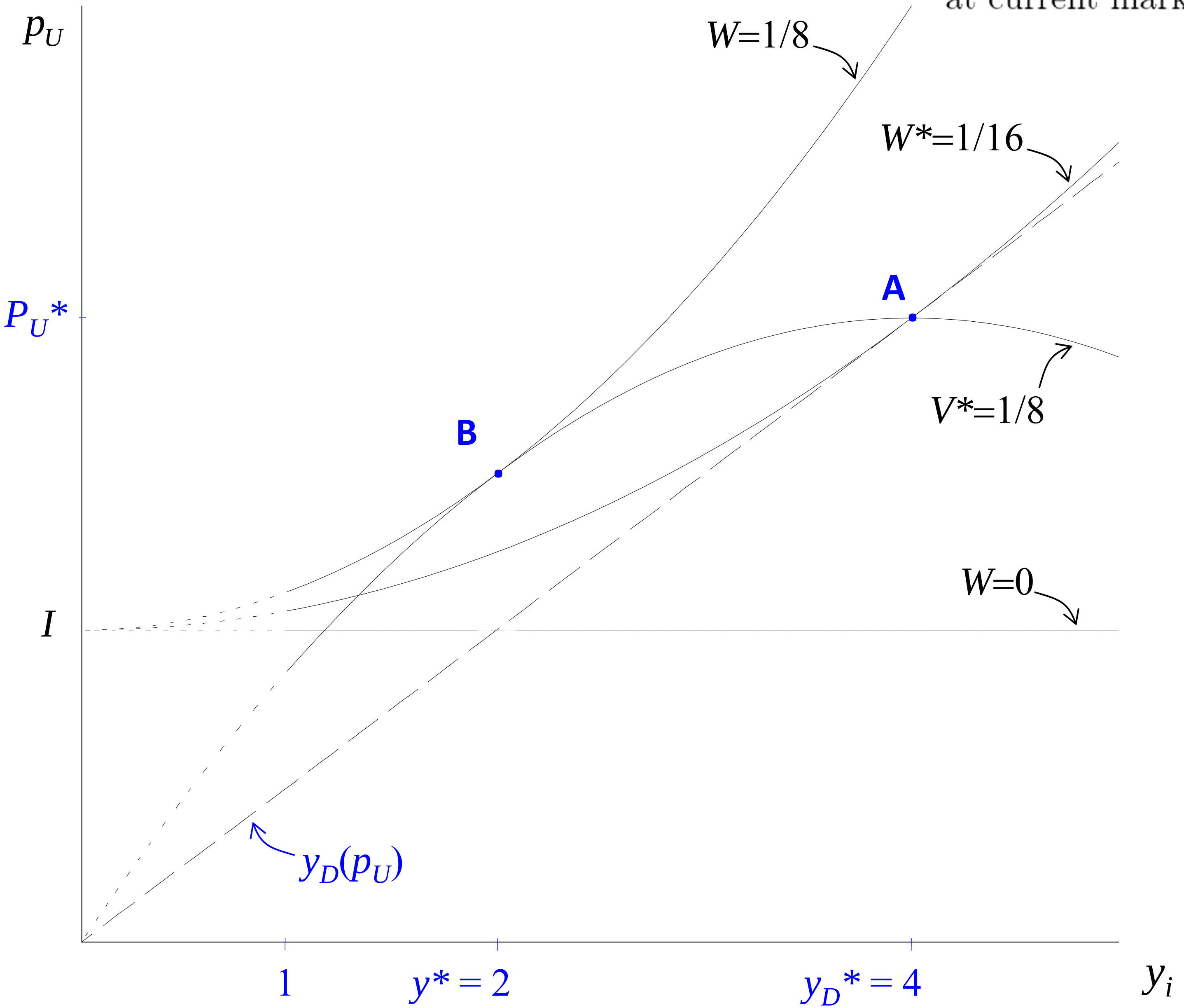
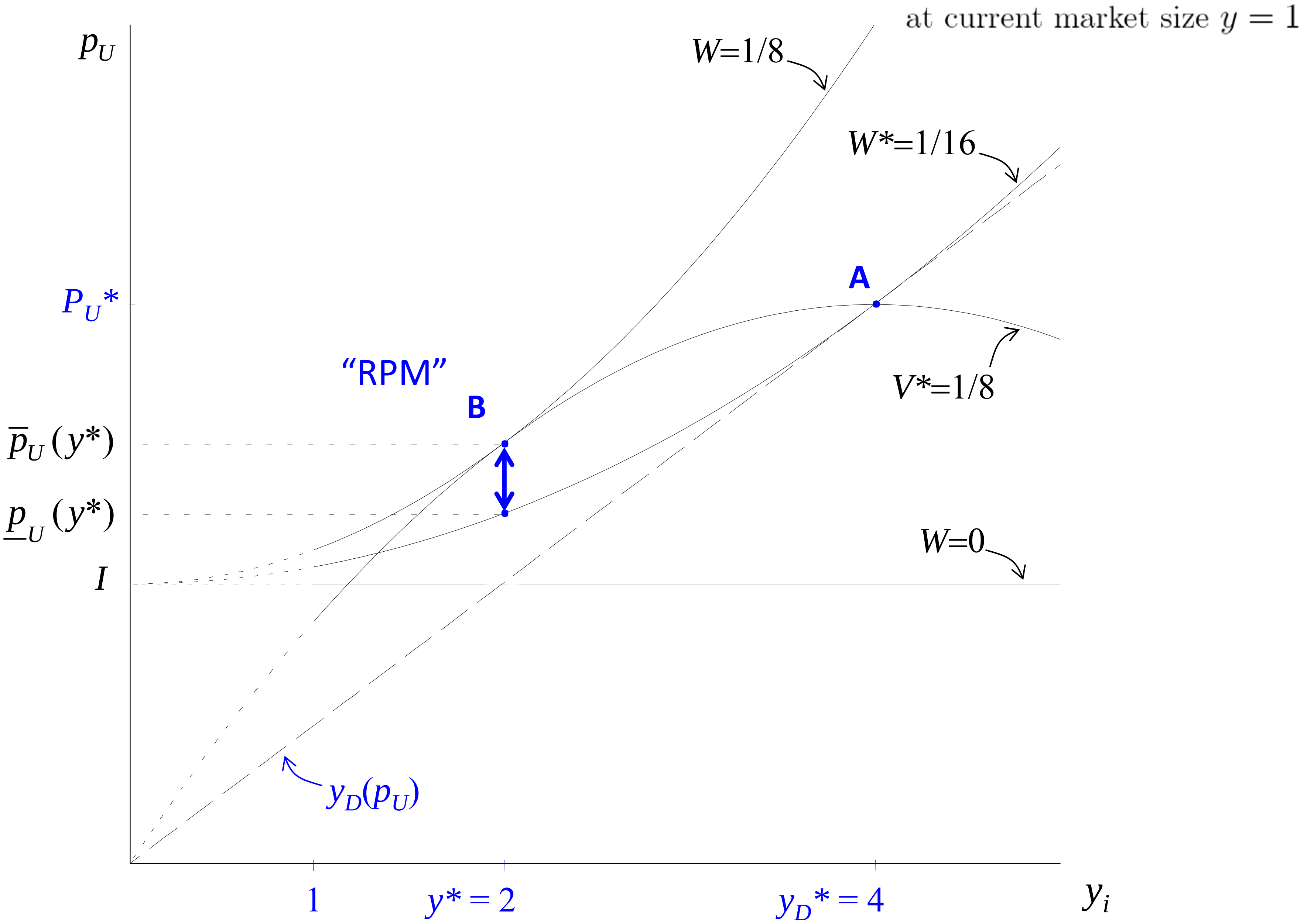
all $y \leq y^*$, where $\underline{p}_U\left(y\right)$ is defined by $W\left(y, y_D^*, p_U^*\right) = W\left(y, y_i, \underline{p}_U\left(y\right)\right)$, and $\overline{p}_U\left(y\right)$ by $V\left(y, y_D^*, p_U^*\right) = V\left(y, y_i, \overline{p}_U\left(y\right)\right)$.

Proposition 4 *Suppose that* $y \leq y^*$*. In a contract analogous to resale price maintenance, the upstream firm chooses the investment trigger* y^* *and charges the input price* $\overline{p}_U\left(y^*\right)$*, as defined by* $V\left(y, y_D^*, p_U^*\right) = V\left(y, y^*, \overline{p}_U\left(y^*\right)\right)$*. The downstream value is the same as in the separation outcome, and the upstream value is* $W\left(y, y^*, \overline{p}_U\left(y^*\right)\right) > W\left(y, y_D^*, p_U^*\right)$*.*

at current market size $y = 1$




at current market size $y = 1$
p_U
W=1/8
W^*=1/16
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A
"RPM"
B
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$\overline{p}_U(y^*)$
$\underline{p}_U(y^*)$
W=0
I
$y_D(p_U)$
1
$y^* = 2$
$y_D^* = 4$
y_i

Analogous to TPT upstream problem:

$$\max_{p_U,t_U} W\left(y,p_U\right)+t_U$$

$$\text{s.t.}\quad V\left(y,y_D\left(p_U\right),p_U\right)-t_U\geq V\left(y,y_D^*,p_U^*\right),$$

$$W\left(y,p_U\right)+t_U\geq W\left(y_D^*,p_U^*\right),$$

all $y\leq y^*$.

Analogous to TPT upstream problem:

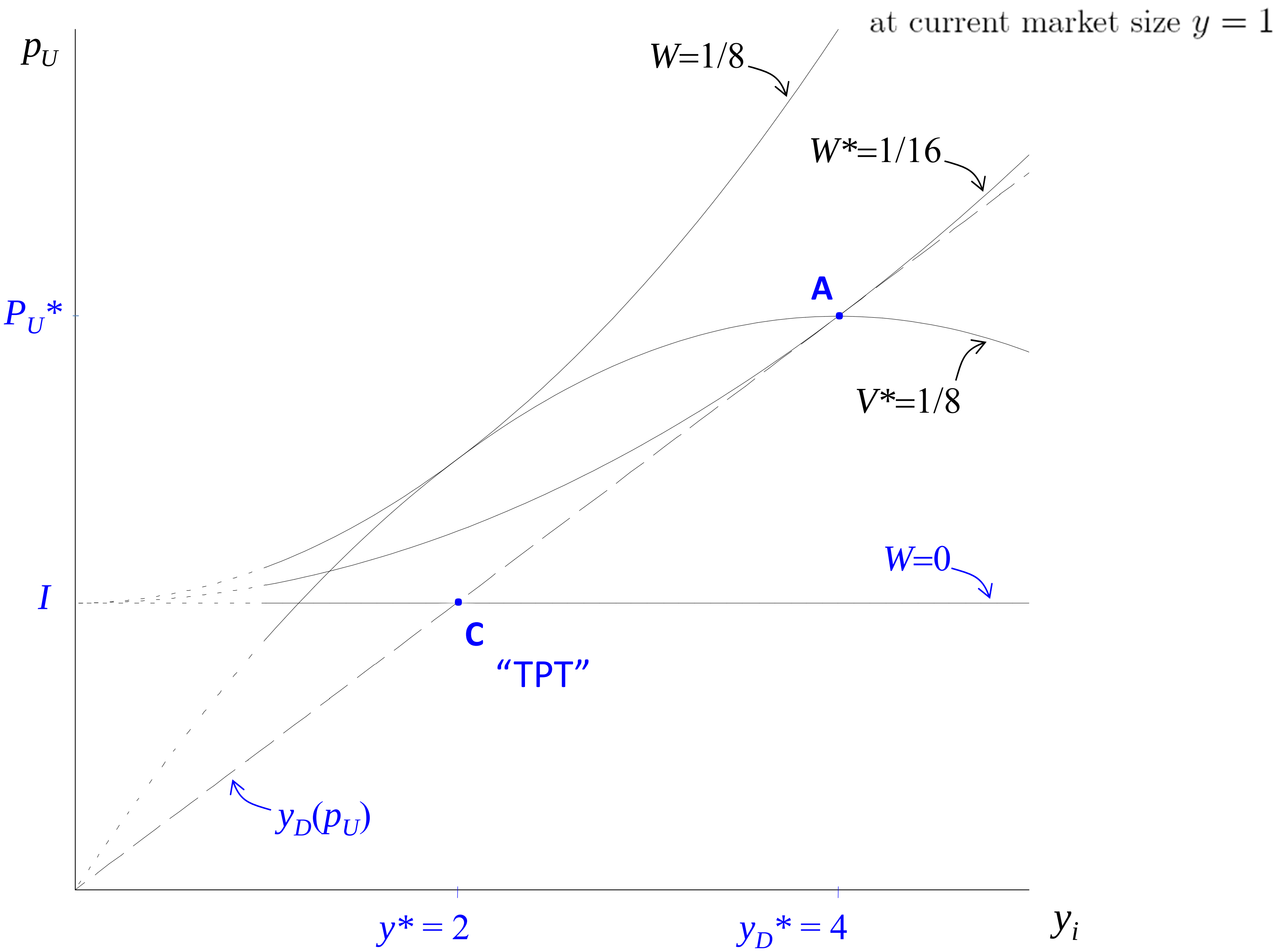
$$\max_{p_U, t_U} W(y, p_U) + t_U$$

$$\text{s.t.} \quad V(y, y_D(p_U), p_U) - t_U \geq V(y, y_D^*, p_U^*),$$

$$W(y, p_U) + t_U \geq W(y_D^*, p_U^*),$$

all $y \leq y^*$.

Proposition 5 *Suppose that* $y \leq y^*$. *In a contract analogous to a two-part tariff, the upstream firm charges the price* I, *and chooses the transfer* $t_U^*(y) = V(y, y^*, I) - V(y, y_D^*, I)$. *The downstream value is the same as in the separation outcome, and the upstream value is* $W(y^*, I) + t_U^*(y) > W(y, y_D^*, p_U^*)$.


at current market size $y = 1$
p_U
W=1/8
W*=1/16
A
P_U*
V*=1/8
I
W=0
C
"TPT"
$y_D(p_U)$
$y^* = 2$
$y_D^* = 4$
y_i

The value of a follower that enters at a threshold y_F and pays a price p_U is

$$\boxed{F\left(y, y_F, p_{U_F}\right) = \left(\frac{y}{y_F}\right)^{\beta}\left(\frac{\pi_D}{r-\alpha}y_F - p_{U_F}\right)}$$

all $y \leq y_F$. The optimal second spot price for the upstream firm is

$$\boxed{p^*_{U_F} = \frac{\beta}{\beta-1}I = p^*_U}$$

same price as with separation with a single buyer, with delayed investment

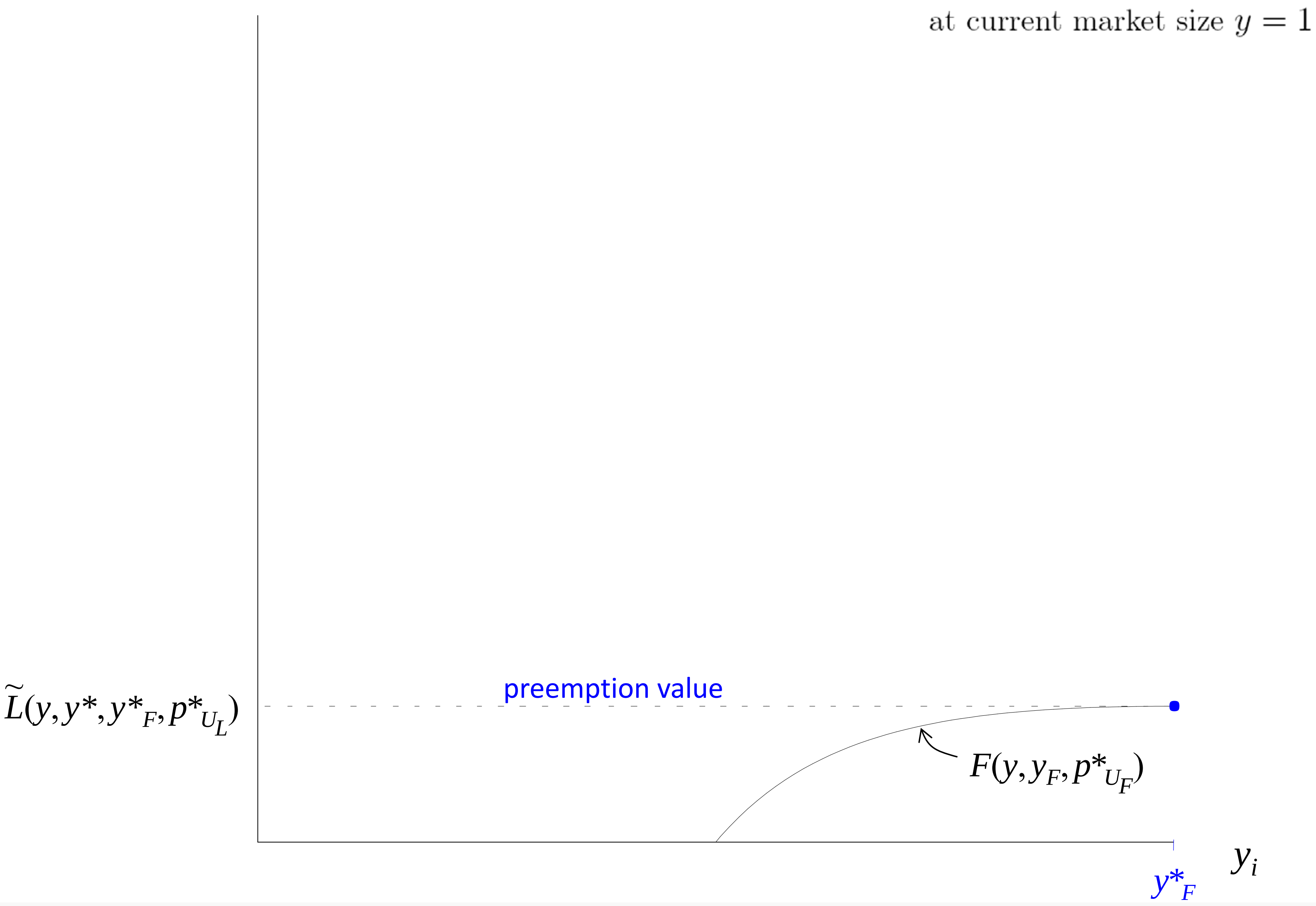
The optimal follower entry threshold is

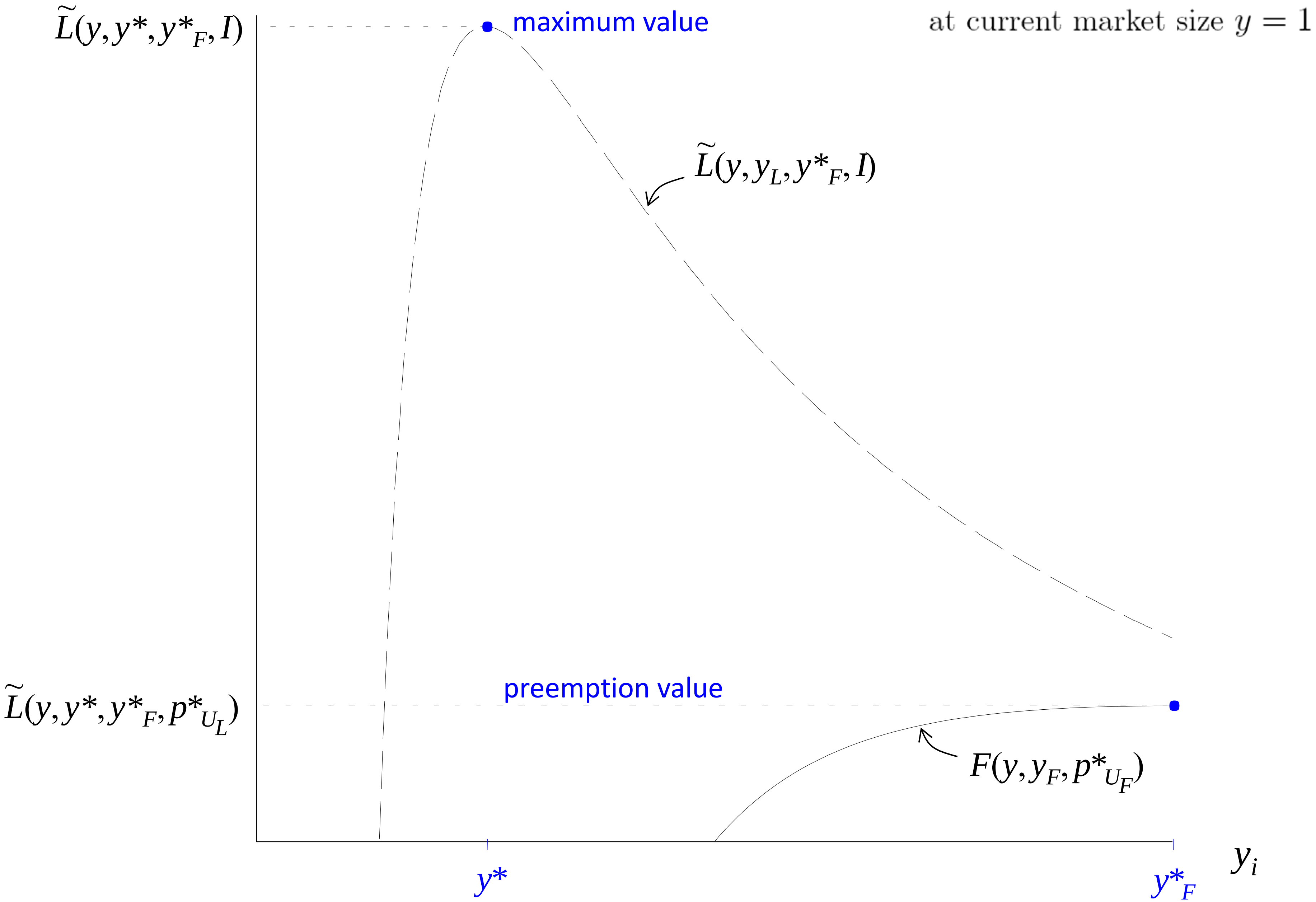
$$\boxed{y^*_F = \left(\frac{\beta}{\beta-1}\right)^2 \frac{r-\alpha}{\pi_D}I > y^*_D = \left(\frac{\beta}{\beta-1}\right)^2 \frac{r-\alpha}{\pi_M}I}$$

The value of a firm that invests immediately at the current market size y, given that its rival invests optimally as a follower at y^*_F, is

$$\boxed{L\left(y, p_{U_L}\right) = \frac{\pi_M}{r-\alpha}y - p_{U_L} - \left(\frac{y}{y^*_F}\right)^{\beta}\frac{\pi_M - \pi_D}{r-\alpha}y^*_F}$$

all $y \leq y_F$.


at current market size $y = 1$
$\widetilde{L}(y, y^*, y^*_F, p^*_{U_L})$
preemption value
$F(y, y_F, p^*_{U_F})$
y_i
y^*_F


$\widetilde{L}(y, y^*, y^*_F, I)$
maximum value
at current market size $y = 1$
$\widetilde{L}(y, y_L, y^*_F, I)$
$\widetilde{L}(y, y^*, y^*_F, p^*_{U_L})$
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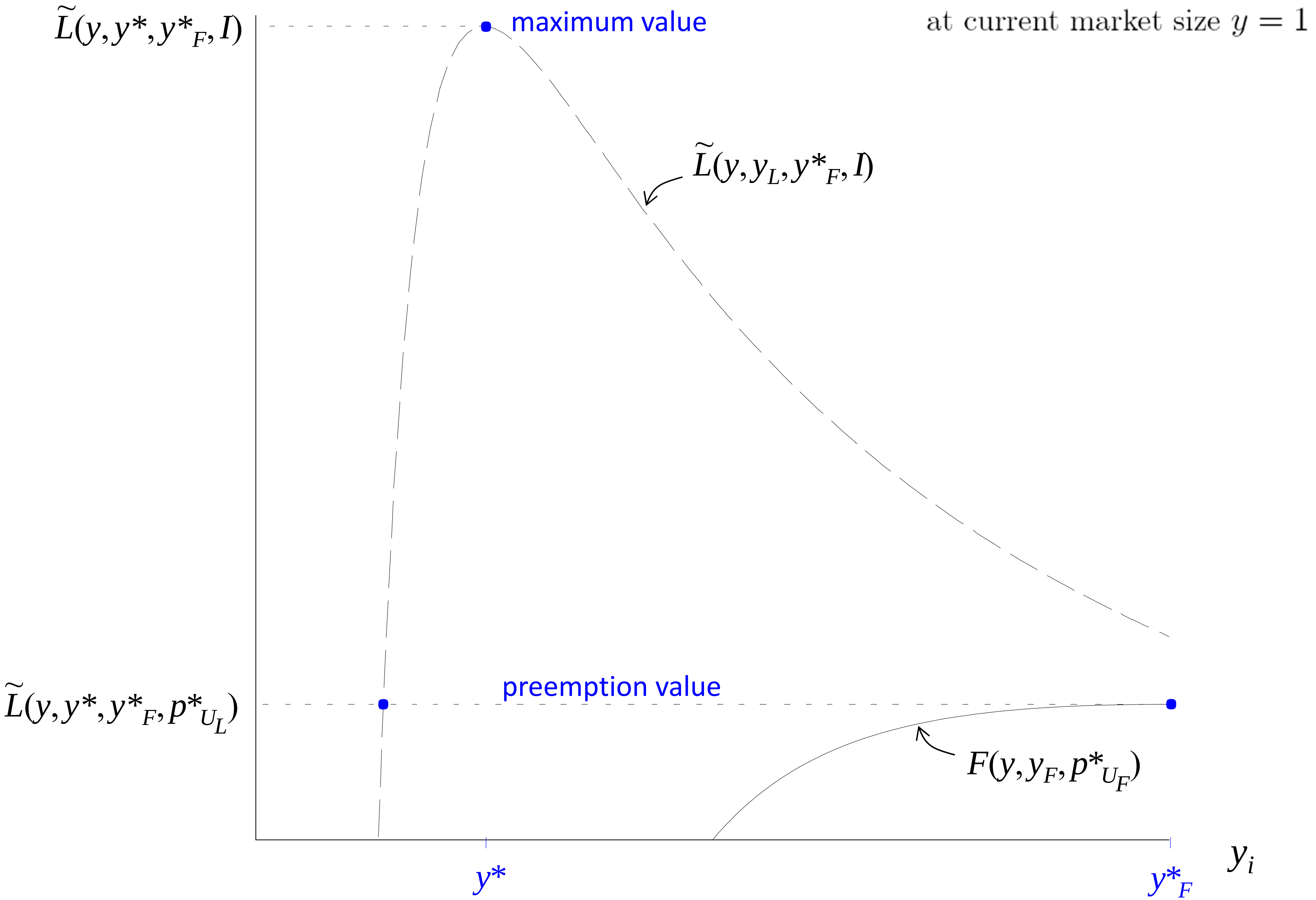
y_i
y^*
y^*_F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y^*_F
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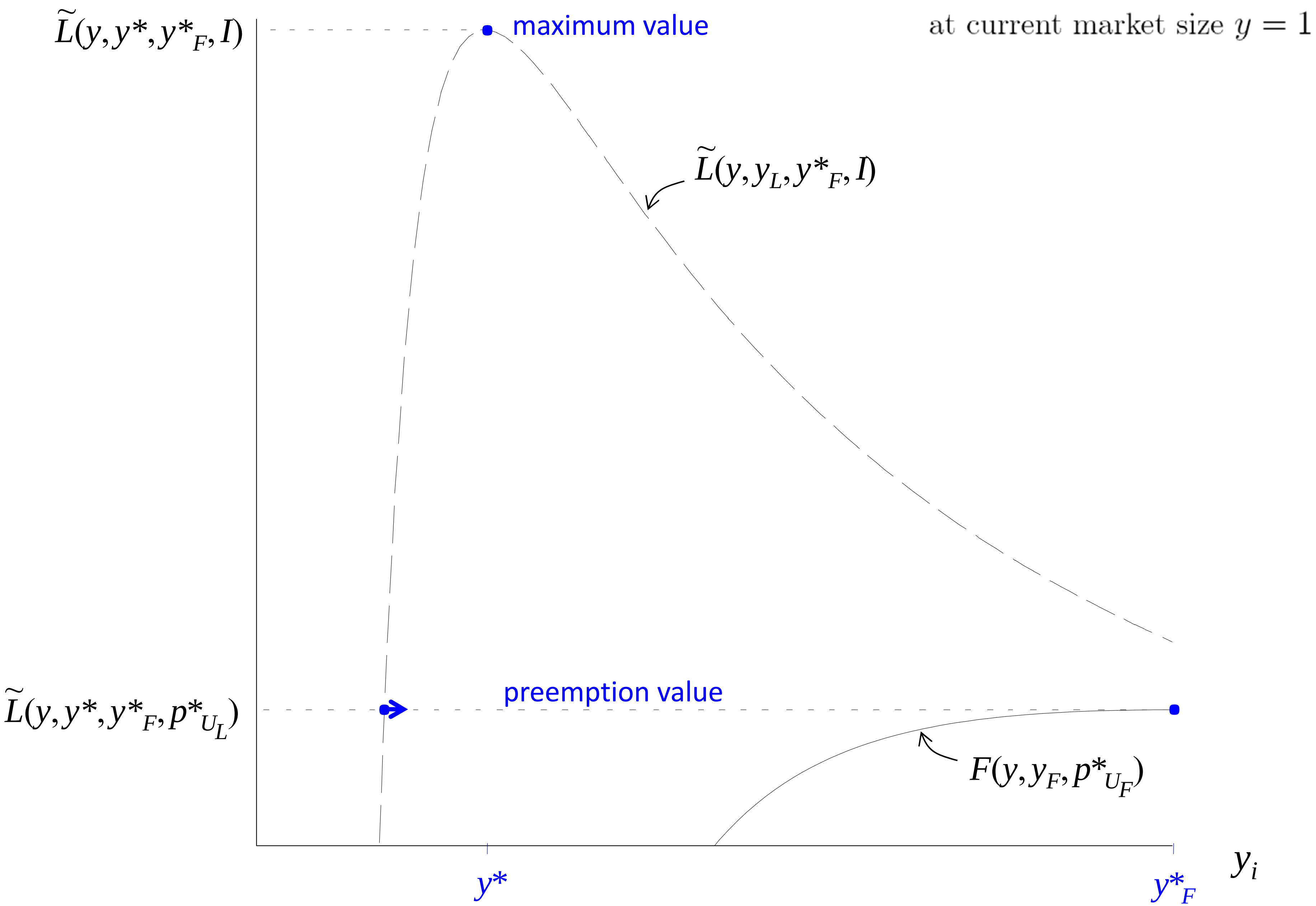
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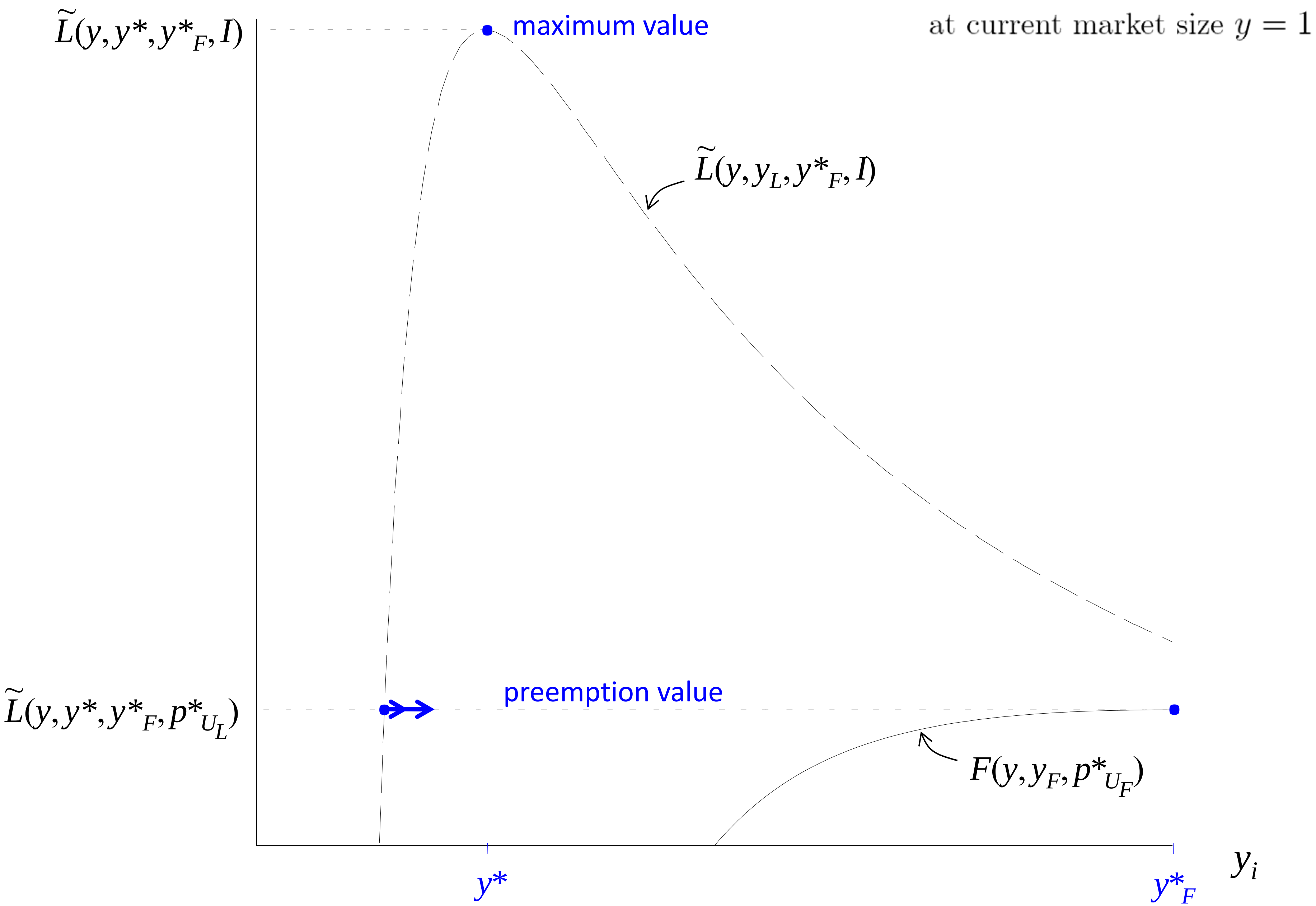
y_i
y^*
y^*_F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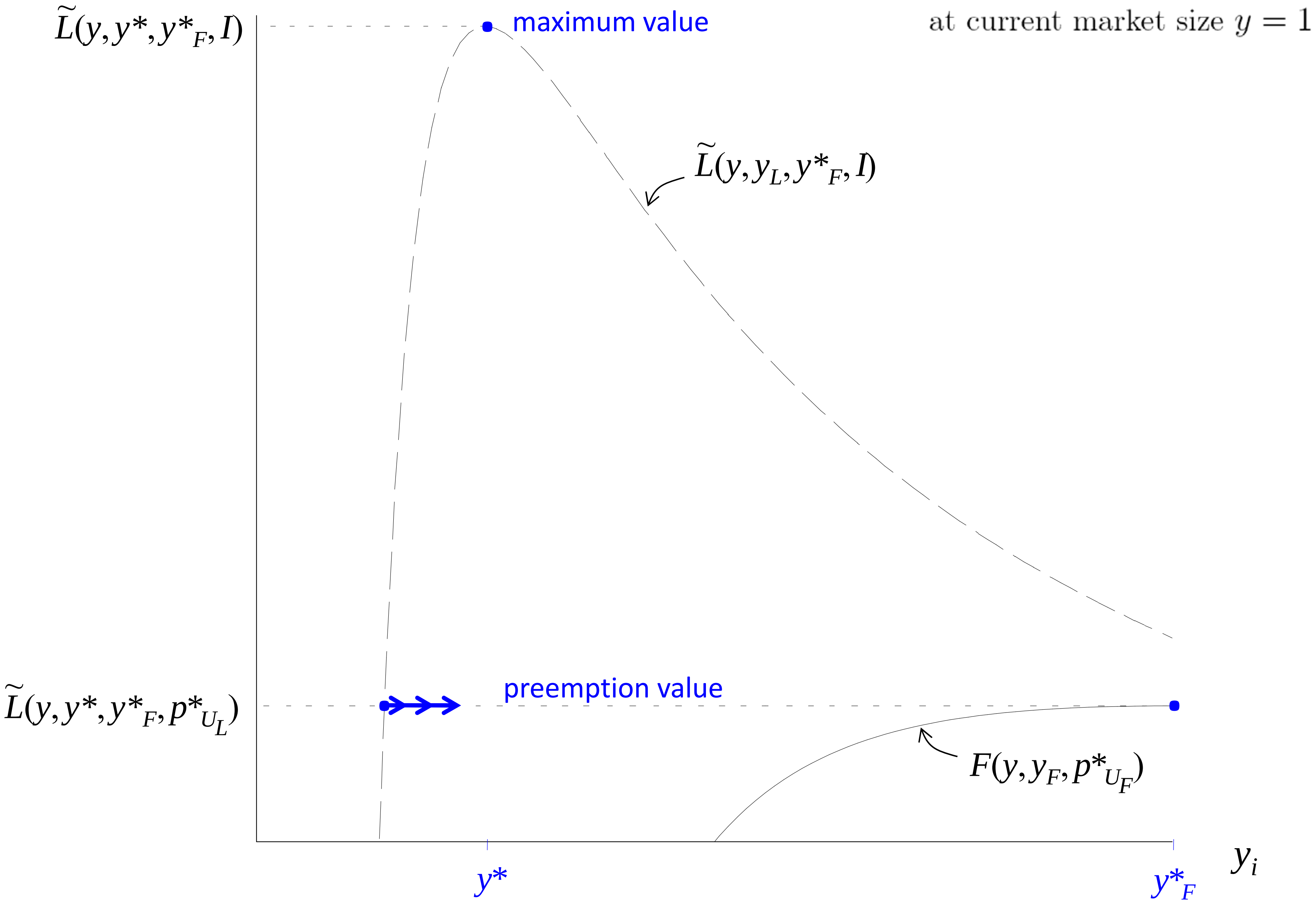
y^*
y^*_F
y_i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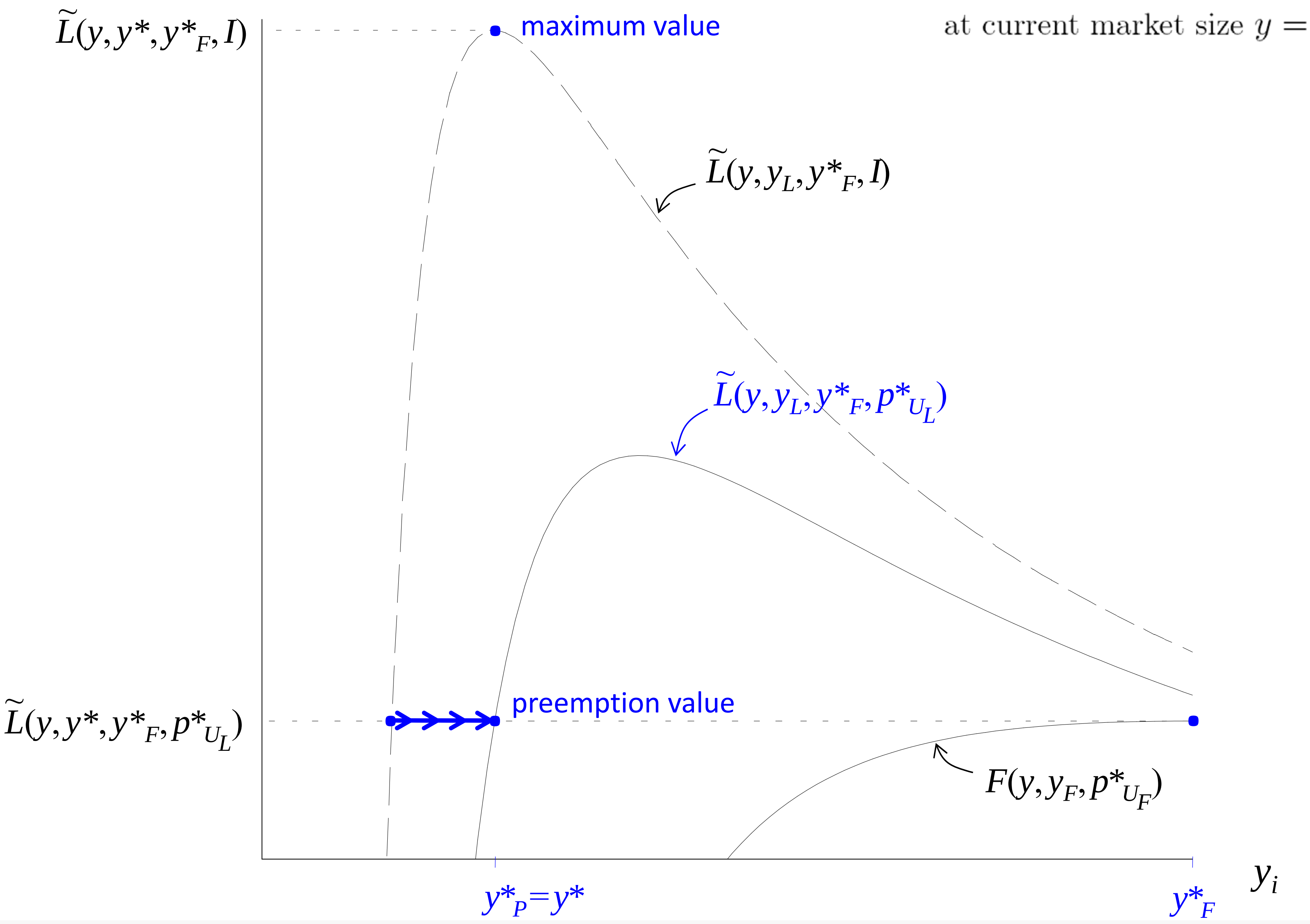
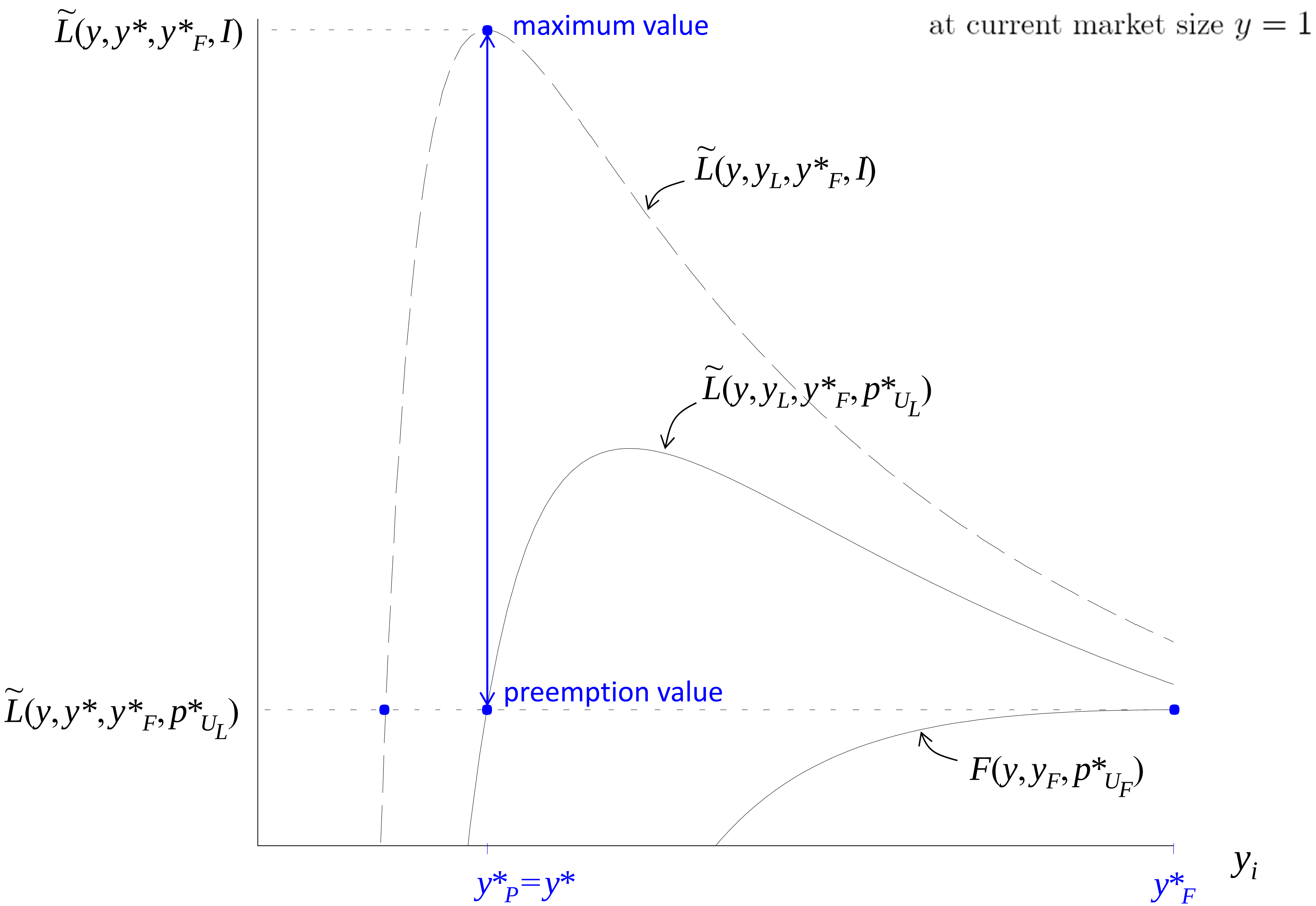
$F(y, y_F, p^*_{U_F})$
y^*
y^*_F
y_i



at current market size $y = 1$


$\widetilde{L}(y, y^*, y^*_F, I)$
maximum value
at current market size $y = 1$
$\widetilde{L}(y, y_L, y^*_F, I)$
$\widetilde{L}(y, y_L, y^*_F, p^*_{U_L})$
$\widetilde{L}(y, y^*, y^*_F, p^*_{U_L})$
preemption value
$F(y, y_F, p^*_{U_F})$
y_i
$y^*_P = y^*$
y^*_F

Proposition 6 *In the separated case with two downstream firms, there is a unique equilibrium characterized by:*

$$(i)\ \textit{downstream triggers} \quad : \quad y_P^* = \frac{\beta}{\beta - 1}\frac{r-\alpha}{\pi_M}I \ = \ y^* \ < \ y_F^* = \left(\frac{\beta}{\beta-1}\right)^2 \frac{r-\alpha}{\pi_D}I$$

Downstream competition (with only two firms) restores private efficiency, in that the leader invests at the same time as in the benchmark integrated structure.

Proposition 6 *In the separated case with two downstream firms, there is a unique equilibrium characterized by:*

$$(i)\ \textit{downstream triggers} \quad : \quad y_P^* = \frac{\beta}{\beta-1}\frac{r-\alpha}{\pi_M}I \ = \ y^* \ < \ y_F^* = \left(\frac{\beta}{\beta-1}\right)^2 \frac{r-\alpha}{\pi_D}I$$

$$(ii)\ \textit{upstream prices} \quad : \quad p_{U_L}^* = \left(1-\Gamma\left(\beta, \frac{\pi_M}{\pi_D}\right)\right)\frac{\beta}{\beta-1}I \ < \ p_{U_F}^* = \frac{\beta}{\beta-1}I \ = \ p_U^*$$

Downstream competition (with only two firms) restores private efficiency, in that the leader invests at the same time as in the benchmark integrated structure.

The leader pays a lower price than with integration. The follower, which invests later, pays the same price as in the separated case with only one buyer.

Sketch of proof: The threshold $y_P\left(p_{U_L}\right)$ verifies $L\left(y_P\left(p_{U_L}\right), p_{U_L}\right) = F\left(y_P\left(p_{U_L}\right), y_F^*, p_{U_F}^*\right)$. Specifically, $y_P\left(p_{U_L}\right)$ is defined implicitly by:

$$\frac{\pi_M}{r-\alpha} y_P - p_{U_L} - \left(\frac{y_P}{y_F^*}\right)^\beta \gamma \left(\gamma \frac{\pi_M}{\pi_D} - 1\right) I = 0. \tag{15}$$

The decision problem of the upstream firm can then be examined. Its value when the current market size is y is:

$$\tilde{W}\left(y, p_{U_L}, p_{U_F}^*\right) = \left(\frac{y}{y_P(p_{U_L})}\right)^\beta \left(p_{U_L} - I\right) + \left(\frac{y}{y_F^*}\right)^\beta \left(p_{U_F}^* - I\right). \tag{16}$$

From (15) we obtain an expression of $p_{U_L} - I$ that we plug into (16), to find:

$$\tilde{W}\left(y, p_{U_L}, p_{U_F}^*\right) = V\left(y, y_P, I\right) + U\left(y, p_{U_F}^*\right),$$

where $U\left(y, p_{U_F}^*\right)$ is independent of y_P, and $V\left(y, y_P, I\right)$ is the integrated payoff of the integrated case. ■

Proposition 7 *In a preemption equilibrium, downstream triggers and upstream prices satisfy the following rankings:*

$$y_P^* = y^* < y_D^* < y_F^* \text{ and } I < p_{U_L}^* < p_{U_F}^* = p_U^*.$$

Moreover, a higher market growth rate, and lower volatility, result in higher triggers $\{y_P^*, y_F^*\}$ *and higher downstream prices* $\left\{p_{U_L}^*, p_{U_F}^*\right\}$ *with:*

$$\varepsilon_{y_F^*/\beta} < \varepsilon_{y_P^*/\beta} < 0 \text{ and } \varepsilon_{p_{U_F}^*/\beta} < \varepsilon_{p_{U_L}^*/\beta} < 0.$$

$$\tilde{W}\left(y, p_{U_L}^*, p_{U_F}^*\right)$$

$$= \underbrace{\left(\frac{y}{y_P^*}\right)^{\beta}\left(p_{U_L}^* - I\right)}_{\lesseqgtr (?)\, W\left(y, p_U^*\right)} + \underbrace{\left(\frac{y}{y_F^*}\right)^{\beta}\left(p_{U_F}^* - I\right)}_{< W\left(y, p_U^*\right)}$$

$$\tilde{V}\left(y, p_{U_L}^*, p_{U_F}^*\right)$$

$$= \underbrace{\frac{1}{2}\left[\left(\frac{y}{y_P^*}\right)^{\beta}\left(\frac{\pi_M}{r-\alpha}y_P^* - p_{U_L}^*\right) + \left(\frac{y}{y_F^*}\right)^{\beta}\left(\frac{\pi_D - \pi_M}{r-\alpha}y_F^*\right)\right]}_{\lesseqgtr (?)\, V\left(y, y_D^*, p_U^*\right)} + \frac{1}{2}\underbrace{\left(\frac{y}{y_F^*}\right)^{\beta}\left(\frac{\pi_D}{r-\alpha}y_F^* - p_{U_F}^*\right)}_{< V\left(y, y_D^*, p_U^*\right)}$$

Proposition 8 *For all* $y > 0$, *for all* β *and all* $\pi_D < \pi_M$, *downstream value is lower and upstream value is higher in a preemption equilibrium than under bilateral monopoly:*

$$\tilde{V}\left(y, p^*_{U_L}, p^*_{U_F}\right) < V\left(y, y^*_D, p^*_U\right) \text{ and } W\left(y, p^*_U\right) < \tilde{W}\left(y, p^*_{U_L}, p^*_{U_F}\right).$$

Moreover, for large enough $\left\{\beta, \frac{\pi_M}{\pi_D}\right\}$, *total industry value is greater in a preemption equilibrium than under bilateral monopoly.*

1. Motivation
2. The Model
3. Integration/Separation
4. Vertical Restraints
5. Two Buyers
6. ▶ Final Remarks

Now

- vertical contract with upstream supplier as a *pre*-commitment to value maximization
- less market power upstream results in *privately* sub-optimal investment sequence

Next

- learning effects may decrease the production cost for the second input supplied
- relax the assumption of a geometric Brownian motion (jumps)
- more market power for downstream firms

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