Optimal Collusion with Limited Liability and Policy Implications

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Objectives

Our objectives are:

1) to characterize the ability of oligopolistic firms to implement a collusive strategy when their ability to punish deviations over one or several periods is limited;

2) to draw policy implications.

The limited liability constraint formalizes:

- structural conditions (*e.g.*, finite demand);

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The limited liability constraint formalizes:

- structural conditions (*e.g.*, finite demand);
- a regulatory mechanism (*e.g.*, prudential ratio);
- financial market pressure (*e.g.*, profitability target).

The Main Results

When the limited liability constraint binds:

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- this discount threshold is *strictly* lower with a multi-period punishment profile than with a single-period punishment scheme;
- a longer punishment is only an *imperfect* substitute for more immediate severity.

As a policy implication, a well-adjusted limited liability constraint can restore competition by iteration.

Symmetric firms in $N = \{1, \ldots, n\}$ play a repeated stage-game over $t = 1, 2, \ldots, \infty$.

Initially all firms play $a_m \in A \subset R_+$ for an individual payoff $\pi_m \equiv \pi(a_m)$ in each period.

- If a firm deviates from a_m in period t, all firms switch to the punishment action $a_{P,k}$, and earn $\pi(a_{P,k}) \leq \pi_m$, in period(s) $t+1, \ldots, t+k, \ldots,\ t+l$, with $l \geq 1$.
- If a firm deviates from $a_{P,k}$ in period $t+k$, with $k = 1, \ldots, l$, the l-period punishment phase restarts.

Otherwise after l punishment periods all firms switch back to a_m.

The Model

The main assumptions:

- $(A1)$ Firms incur a fixed cost $f \geq 0$, and a variable cost $c(q_i) \geq 0$, to sell substitutable goods (possibly differentiated), with either price $(a = p)$ or quantity $(a = q)$ as a strategic variable;
- $(A2)$ Each firm i's inverse demand function $p_i : R_+^n \to R_+$ is non-increasing and continuous;
- $(A3)$ $p_i(\mathbf{0}) > c$ and $\lim_{q_i \to \infty} p_i(q_i, \mathbf{q}_{-i}) = 0$, any $\mathbf{q}_{-i}$ in R_+^{n-1}.

To compare, the main assumptions in Abreu (1986) are:

- $(\widetilde{A}1)$ Firms sell a homogeneous good at constant marginal cost $c > 0$, and their strategic variable is quantity;
- $(\widetilde{A}2)$ The market inverse demand function $p(q) : R_+ \to R_+$ is *strictly* decreasing and continuous in $q = \sum_{i \in N} q_i$;
- $(\widetilde{A}3)$ $p(0) > c$ and $\lim_{q \to \infty} p(q) = 0$.

$$\Rightarrow \lim_{q_i \to \infty} (p(q) - c)\, q_i = -\infty$$

The Model

The notation:

a_m : collusive action

a_{NE} : stage-game Nash equilibrium action

$a_{P,k}$: punishment action in period $k = 1, \ldots, l$

$\pi(a)$: stage profit when all firms play the same a (with $\pi_m \equiv \pi(a_m)$)

$\pi_i^d(a)$: firm i's one-shot best reply benefits to a as played by all rivals in $N \backslash \{i\}$

$\delta \in (0,1)$: discount factor

The Benchmark

Incentive constraint (no deviation from collusion):

$$\pi_i^d(a_m) - \pi_m \leq \delta\left[\pi_m - \pi(a_P)\right] \qquad (IC0)$$

Incentive constraint (no deviation from punishment):

$$\pi_i^d(a_P) - \pi(a_P) \leq \delta\left[\pi_m - \pi(a_P)\right] \qquad (IC1)$$

Participation constraint:

$$(1-\delta)\left[\pi_m - \pi(a_P)\right] \leq \pi_m \qquad (PC)$$

Limited Liability constraint:

$$\pi(a_P) \geq \underline{\pi}, \qquad (LLC)$$

with $\underline{\pi} \equiv \pi\left(\underline{a}_P\right)$.

The Benchmark

Given a_m, the single-period punisment δ-minimization problem in a_P is

$$\min_{a_P \in A} \delta$$
$$s.t. \quad IC0,\ IC1,\ PC,\ LLC$$

The Benchmark

Proposition 1

The collusive action $a_m \preceq a_m^*$ is implementable with a single-period punishment if and only if $\delta \geq \delta_1^*$, with

$$\delta_1^* = \begin{cases} \delta^* \equiv \frac{\pi_i^d(a_m) - \pi_m}{\pi_m - \pi(a_P^*)} & \text{if } a_P^* \succeq \underline{a}_P, \overline{a}_P \quad \text{(regime 1);} \\ \overline{\delta} \equiv \frac{\pi_i^d(a_m) - \pi_m}{\pi_m - \overline{\pi}} & \text{if } \overline{a}_P \succeq \underline{a}_P, a_P^* \quad \text{(regime 2);} \\ \underline{\delta} \equiv \frac{\pi_i^d(a_m) - \pi_m}{\pi_m - \underline{\pi}} & \text{if } \underline{a}_P \succeq a_P^*, \overline{a}_P \quad \text{(regime 3).} \end{cases} \tag{1}$$

If $\underline{\pi} > \pi_m - \left(\pi_i^d(a_m) - \pi_m\right)$ then $\underline{\delta} > 1$ and a_m cannot be implemented for any δ.

Remark 1

If $a_P^* \succeq \underline{a}_P, \overline{a}_P$, so that regime 1 applies, $\delta^* \geq \overline{\delta}, \underline{\delta}$.

The Multi-Period Setup

If a firm does *not* deviate from the punishment path, the continuation profits it earns from period $s+1$ onward is

$$V_s\left(\mathbf{a}_P, \delta\right) = \sum_{k=s+1}^{l} \delta^{k-s-1} \pi(a_{P,k}) + \sum_{k=l+1}^{\infty} \delta^{k-s-1} \pi_m .$$

Multi-period Incentive constraints:

$$\pi_i^d(a_m) - \pi_m \leq \delta\left[V_0(\mathbf{a}_m, \delta) - V_0(\mathbf{a}_P, \delta)\right], \qquad (MIC0)$$

and

$$\pi_i^d\left(a_{P,1}\right) - \pi\left(a_{P,1}\right) \leq \delta\left[V_1(\mathbf{a}_P, \delta) - V_0(\mathbf{a}_P, \delta)\right],$$

$$\ldots$$

$$\pi_i^d\left(a_{P,s}\right) - \pi(a_{P,s}) \leq \delta\left[V_s(\mathbf{a}_P, \delta) - V_0(\mathbf{a}_P, \delta)\right], \qquad (MIC1, ..., l)$$

$$\ldots$$

$$\pi_i^d\left(a_{P,l}\right) - \pi(a_{P,l}) \leq \delta\left[V_l(\mathbf{a}_P, \delta) - V_0(\mathbf{a}_P, \delta)\right],$$

with $1 \leq s \leq l$.

The Multi-Period Setup

Multi-period Participation constraint:

$$(1-\delta)\left[V_0\left(\mathbf{a}_m,\delta\right)-V_s\left(\mathbf{a}_P,\delta\right)\right]\leq\pi_m, \tag{MPC}$$

all $s=0,1,\ldots,l$.

Multi-period Limited Liability constraint:

$$\pi(a_{P,k})\geq\underline{\pi}, \tag{MLLC}$$

with $1\leq k\leq l$, all $l\geq 2$, and $\underline{\pi}\equiv\pi\left(\underline{a}_P\right)$.

The Multi-Period Setup

Given all constraints, the multi-period punishment problem is

$$\begin{array}{ll} \min\limits_{(a_{P,1},\ldots,a_{P,l})\in A^{l}} & \delta \\ s.t. & (MIC\,0 - MIC\,l); MPC; MLLC \end{array}$$

The Multi-Period Setup

Lemma 2

Given $a_{P,1}$, the lowest discount factor δ verifying $(MIC\,0)$ and $(MIC\,1)$ results from punishment actions $a_{P,k}$, with $k > 1$, such that these two multi-period incentive constraints bind.

Proposition 2

In the multi-period punishment scheme, if $a_P^* \succeq \overline{a}_P, \underline{a}_P$ the collusive action $a_m \preceq a_m^*$ is implementable if and only if $\delta \geq \delta^*$, and $\mathbf{a}_P^* \equiv (a_P^*, a_m, \ldots, a_m)$ is optimal.

Lemma 3

The lowest δ compatible with $(MIC\,0)$ and (MPC) is $\overline{\delta} \equiv \frac{\pi_i^d(a_m)-\pi_m}{\pi_i^d(a_m)}$.

Proposition 3

In the multi-period punishment scheme, if $\overline{a}_P \succeq \underline{a}_P, a_P^*$, the collusive action $a_m \preceq a_m^*$ is implementable if and only if $\delta \geq \overline{\delta}$, and $\overline{\mathbf{a}}_P \equiv (\overline{a}_P, a_m, \ldots, a_m)$ is optimal.

The Multi-Period Setup

Lemma 4

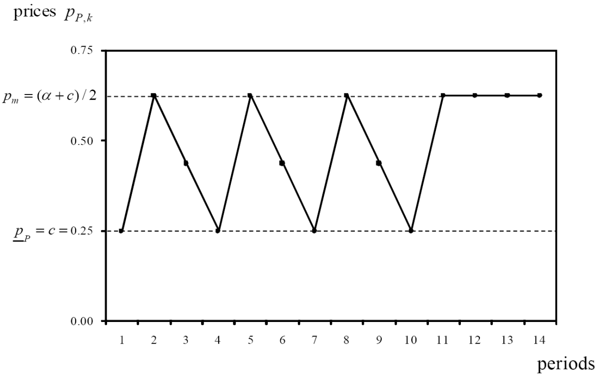
The lowest δ *compatible with* $(MIC\,0)$ *and* $(MLLC)$ *is* $\underline{\delta}' \equiv \frac{\pi_i^d(a_m)-\pi_m}{\pi_i^d(a_m)-\pi_i^d(\underline{a}_P)}$.

Proposition 4

In the multi-period punishment scheme, if $\underline{a}_P \succeq a_P^*, \overline{a}_P$ *collusion at* a_m *is implementable if and only if* $\delta \geq \underline{\delta}_M \equiv \sup\{\overline{\delta}, \underline{\delta}'\}$, *with* $\underline{\mathbf{a}}_P \equiv (\underline{a}_P, a_{P,2}, \ldots, a_{P,l})$ *of finite length* l.

Remark 4

If $(MLLC)$ is strictly binding, that is if $\underline{a}_P \succ a_P^*, \overline{a}_P$, there exits a continuum of optimal punishments $(\underline{a}_P, a_2, \dots, a_l)$ of finite length $l \geq 2$, s.t. a_m is implementable for $\delta = \underline{\delta}_M$.



Proposition 5

If $\underline{a}_P \succ a_P^*, \overline{a}_P$, and additional punishment periods are introduced, the lowest discount factor $\underline{\delta}_M$ that permits the implementation of $a_m \preceq a_m^*$ cannot be as low as δ^*, and can attain $\overline{\delta}$ only in particular circumstances. More formally, either $\overline{a}_P \preceq a_P^*$ so that $\delta^* < \underline{\delta}_M < \underline{\delta}$, or $\overline{a}_P \succ a_P^*$ and $\overline{\delta} \leq \underline{\delta}_M < \underline{\delta}$. In the latter case $\underline{\delta}_M = \overline{\delta}$ if and only if $\tilde{a}_P \succeq \underline{a}_P \succ \overline{a}_P \succ a_P^*$.

$\Rightarrow$ $l > 1$ only an *imperfect* substitute to early severe punishment

A Linear Example

Firms in $N = \{1, \ldots, n\}$ incur a constant marginal cost $c \geq 0$ to sell $\mathbf{q}$ to consumers with utility function

$$U(\mathbf{q}, I) = \sum_{i=1}^{n} q_i - \frac{1}{2}\left(\sum_{i=1}^{n} q_i^2 + 2\gamma \sum_{i \neq j} q_i q_j\right) + I,$$

all $q_i, q_j \geq 0$, $j \in N \backslash \{i\}$, where $\gamma \in (0, 1)$ measures product substitutability.

Limited Liability constraint:

$$p_i(\mathbf{q}_P) \geq 0,$$

all $i \in N$.

Proposition 6

The parameter space (c, n, γ) is partitioned in three subsets where either Regime 1, 2, or 3, as defined in (1), applies.

A Linear Example

- *Regime 1 applies if and only if*

  (i) $2 \leq n \leq 3;\ 0 \leq \gamma \leq 1;\ 0 \leq c < 1$; or

  (ii) $4 \leq n \leq 5;\ 0 \leq \gamma \leq 1;\ \underline{c}' \leq c < 1$; or

  (iii) $6 \leq n;\ 0 \leq \gamma \leq \hat{\gamma};\ 0 \leq c < 1$; or

  (iv) $6 \leq n;\ \hat{\gamma} \leq \gamma \leq \check{\gamma};\ \underline{c}' \leq c < 1$.

- *Regime 2 applies if and only if*

  $6 \leq n;\ \check{\gamma} \leq \gamma \leq 1;\ \underline{c}'' \leq c < 1$.

- *Regime 3 applies if and only if*

  (i) $n = 3;\ \gamma = \hat{\gamma} = 1;\ c = \underline{c} = 0$; or

  (ii) $4 \leq n \leq 5;\ \hat{\gamma} \leq \gamma \leq 1;\ 0 \leq c \leq \underline{c}'$; or

  (iii) $6 \leq n;\ \hat{\gamma} \leq \gamma \leq \check{\gamma};\ 0 \leq c \leq \underline{c}'$; or

  (iv) $6 \leq n;\ \check{\gamma} \leq \gamma \leq 1;\ 0 \leq c \leq \underline{c}''$.

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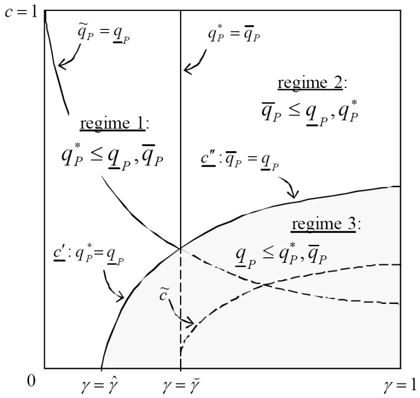

Figure: Collusion regimes in plane (c, γ) for $n \geq 6$. The limited liability constraint binds in the grey area (regime 3). In the benchmark single-period set-up, the collusive quantity is not implementable below the frontier $\tilde{c}$.

Policy Implications

Regulatory constraint:

$$\pi(a) \geq \pi_R, \tag{R}$$

with $\pi_R \equiv \pi(\underline{a}_R)$, and $\underline{a}_P \preceq \underline{a}_R \preceq a_m^*$, implying that $\underline{\pi} \leq \pi_R \leq \pi_m^*$.

Proposition 8

Suppose that firms implement $a_m \succ a_{NE}$. By setting a price floor slightly below the observed transaction price, and by reducing the floor incrementally by iteration, the regulator drives the industry to the stage-game Nash equilibrium a_{NE}.

Policy Implications

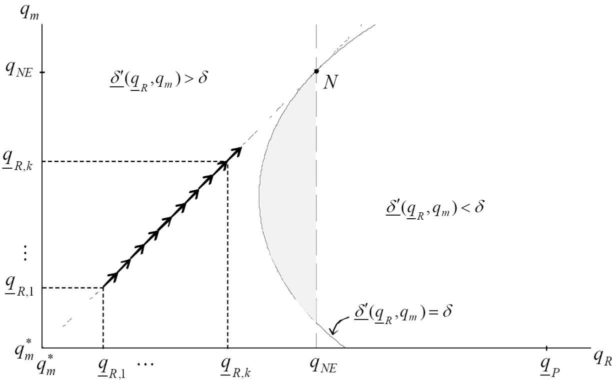

Figure: Cournot linear setup ($n = 5$, $\gamma = 1$, $c = 1/10$, $\delta = 3/5$). Initially, all firms implement q_m^*. A price floor rules out large price reductions (i.e., $q \leq \underline{q}_{R,1}$, with $\underline{q}_{R,1}$ above q_m^*, but only limitedly so). A series of successive adjustments from $\underline{q}_{R,1}$ to $\underline{q}_{R,2}$, $\underline{q}_{R,3}$, ... drives the industry toward the stage-game Nash equilibrium (point N).

Policy Implications

Lemma 5

Consider any implementable collusive action a_m. Then $\underline{a}_R \succ a_{NE}$ implies that $a_m \succ \underline{a}_R$.

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