Dynamic Models in Economics
Problem Set 2:

A Simple Model of Repeated Elections with a Homogenous Electorate

The present exercise is inspired from Ferejohn (1986, *Public Choice*). Many of the activities of office-holding politicians are not directly observable by members of the electorate. Instead, electors are only able to assess the effects of governmental performance on their own well-being. Further, governmental performance is known to depend jointly on the activities of officeholders as well as on a variety of exogenous and essentially probabilistic factors. In other words, the officeholder is an agent of the electorate whose behavior is imperfectly monitored.

Formally, the officeholder observes a random variable, $\theta \in [0, m]$, a subset of the nonnegative real numbers, and then takes an action, $a \in [0, \infty)$, conditioned on that observation. Let F denote the distribution function of θ and assume that it is continuously differentiable. The single-period preferences of the officeholder are written as

$$v(a, \theta) = W - \phi(a),$$

where W is the value of holding office for a single term and ϕ is a positive monotone convex function , and $\phi(0) = 0$.

The representative voter is unable to distinguish the actions of the officeholder from exogenous occurrences. Her single-period preferences are represented as

$$u(a, \theta) = a\theta.$$

Lacking an ability to observe the activities of the incumbent, the elector adopts a simple performance-oriented (or **retrospective**) **voting rule**: *if the utility received at the end of the incumbent's term in office is high enough, she votes to return the incumbent to office; otherwise she removes the incumbent and gives the job to an exogenous challenger.*

Assuming that the elector employs a retrospective voting rule, we can use standard techniques of **dynamic programming** to determine optimal candidate behavior. Once the incumbent has observed a value of θ_t, she will choose an action which maximizes her (discounted) utility from that time onward, assuming that the voter employs a retrospective voting rule with cutoff levels, K_t, K_{t+1}, K_{t+2}, ..., from time t forward. Under the conditions assumed above, this amounts to choosing $a(\theta_t)$ to maximize the present value of utility stream. Obviously, if θ_t is so small that it is not possible to be reelected, then he will choose $a(\theta_t) = 0$. If it is possible to be reelected, then the candidate may choose $a(\theta_t)$ so the reelection constraint is just satisfied: $a(\theta_t) = K_t/\theta_t$. In no event would she be willing to choose any $a(\theta_t)$ larger than the smallest amount that will ensure her reelection.

Questions

Equilibrium Strategies

Let V^I_{t+1} and V^0_{t+1} stand for the expected values of staying in office or leaving office, respectively, given optimal play by voters and candidates from the next election forward, and let δ represent the (common) discount factor employed by all agents.

1. Show that, given the retrospective voting rule $\{K_t\}_{t=0}^{\infty}$, the optimal incumbent stategy is

$$a(\theta_t) = \frac{K_t}{\theta_t} \text{ if and only if } \theta_t \geq \frac{K_t}{\phi^{-1}\left(\delta\left(V^I_{t+1} - V^0_{t+1}\right)\right)}.$$

Interpret this strategy.

2. Show that, if the θ_t are independent, identically distributed variables with c.d.f. $F(\cdot)$ and density $f(\cdot)$, an optimal retrospective voting rule satisfies the following equality:

$$K_t = \frac{[1 - F(\theta^*_t)]}{f(\theta^*_t)}\phi^{-1}\left(\delta\left(V^I_{t+1} - V^0_{t+1}\right)\right),$$

where θ^*_t satisfies $\delta\left(V^I_{t+1} - V^0_{t+1}\right) = \phi(K_t/\theta^*_t)$.

3. Deduce from the previous question that, if the θ_t are independent, uniform, random variables on [0,1], and if $\phi(a) = a$ and $a \in [0,1]$, an optimal retrospective rule must satisfy the following equation:

$$K_t = \min\left\{\frac{1}{2}, \frac{\delta\left(V^I_{t+1} - V^0_{t+1}\right)}{2}\right\}, \text{for all } t.$$

Interpret the last equations.

4. Check that, if $[1 - F(x)]/f(x)$ is a monotone decreasing function, then θ^*_t is independent of δ, t, and W.

5. Deduce from the previous question that, if F is uniform on [0,1], then $\theta^*_t = 1/2$ and $\Pr(\theta_t \geq \theta^*_t) = 1/2$. Interpret.

Alternative Party Systems

Let λ be the probability of obtaining office if the current incumbent is defeated at the next election, which is taken to be exogenously determined. In this interpretation, a pure two-party system corresponds to $\lambda = 1$. At the other extreme, a "pure" multicandidate system would have $\lambda = 0$.

6. Determine the expected value, V_λ^I, of being an incumbent when the voter is playing a stationary retrospective voting strategy with criterion K_λ.

7. Determine the discounted expected utility, V_λ^0, of a candidate out of office when the voter is playing a stationary retrospective voting strategy with criterion K_λ.

8. What is the impact of an increase in λ on the voter's utility?

9. What is the impact of an increase in W on the voter's utility?